Design and Manufacturing of Non-Circular Gears by Given Transfer Function

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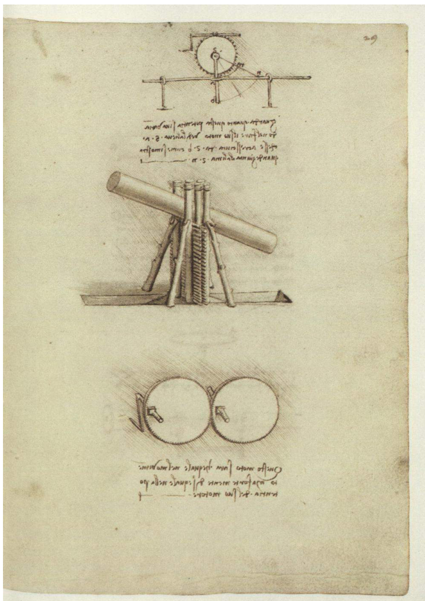
Abstract
Non-circular gears (NCGs) are long known elements of machinery but this days are they unworthy rarely applied mechanisms. For the realization of periodical altering (circular or/and linear) motion the controlled positioning devices are wide-spread. The simplicity, the price, the mechanical power, the tolerance for overaload and the duration of the NCGs are in each case competitve alternative of electircal servo drives.
This paper presents an approach for the calculation of the parameters of NCGs by given transfer functions. The calculation of the rolling curves and of the profiles of teeth are developed in complex algebraic methods. The symbolic demonstrations and the numerical calculations are made by the mathematical software Maple R10. The gears were manufactured by wire EDM technology.

Keywords: non-circular gear, involute, Maple

1. Introduction
Early version of NCGs are shown on the sketches made by Leonardo da Vinchi in the collection of "Codex Madridii" (Fig. 1 a/). The genuine idea of gears with eccentric circle and self-intersected pitch curve dates back to the Leonerdo's drawing book, see Fig. 1 b/. The calculated pitch curves are shown in Fig.2 a/, and the manufactured gears in Fig. 2 b/. ([1-2]).



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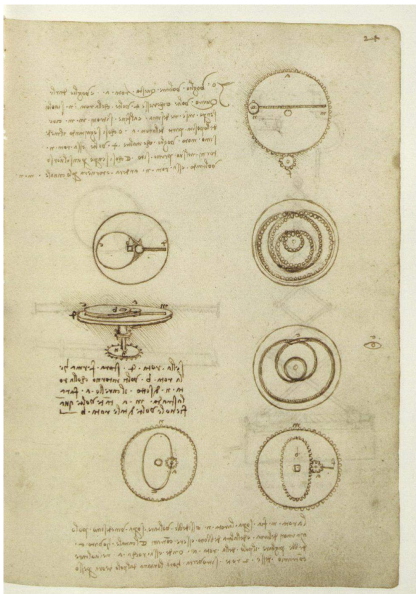


b/

Figure 1

a/ b/

Figure 2

The number of references of NCGs is very limited compared to another technical topics. The most important books in the subject are [3] and [4]. In more recent books ([5], [6]) only few chapter deals with this subject. From the set of Hungarian and international researches the most significant are publications [7-10]. Some publications are abouth NCGs from author of this article [11-15].

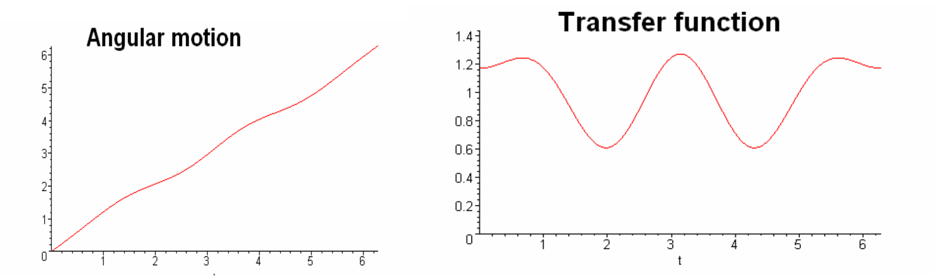
2. The concept of centrodes

Let us give the law of the angular motion of the driven gear[1] in the form of $\Phi = \Phi(t)$ (where "t" is the time parameter). The angular velocity of the driving gear is $\omega_1 = 1$, the distance of the axes are a = const. The centrodes can roll on each other without slipping on the outsides of the curves. The contact point of its is always between the axes.

The η transfer function is the ratio of driven and driving angular velocities, calculated by

$$\eta := \frac{\partial}{\partial t}\Phi \qquad (1)$$

The Figure 3 a/ and b/ illustrates the draws of angular motion and the transfer function for the case of $\Phi := t + \frac{\sin(t)}{7} + \frac{\sin(2\,t)}{9} - \frac{2\sin(3\,t)}{31}$.



a/ b/

Figure 3

While the radii of centrodes R_1 and R_2 is $R_1 + R_2 = a$ and for the pure rolling of it is $\eta = \frac{\omega_2}{\omega_1} = \frac{R_1}{R_2}$ yields

$$R_1 := \frac{a\,\eta}{1+\eta} \qquad R_2 := \frac{a}{1+\eta} \qquad (2)$$

The co-ordinates are of the temporary contact point of the centrodes in the co-ordinate systems fixed at the points of the axes:

$$r_1 := [R_1\cos(t), R_1\sin(t)] \quad r_2 := [R_2\cos(\Phi), R_2\sin(\Phi)] \qquad (3)$$

[1] The driven gear is indexed in the additional by N^o 2, the driving gear by 1

The curvature functions of the centrodes can be obtained by the following form:

$$k_1 := \frac{R_1^{\,2} + 2\left(\frac{\partial}{\partial t} R_1\right)^2 - R_1\left(\frac{\partial^2}{\partial t^2} R_1\right)}{\left(R_1^{\,2} + \left(\frac{\partial}{\partial t} R_1\right)^2\right)^{\left(\frac{3}{2}\right)}} \qquad k_2 := \frac{\left(\frac{\partial}{\partial t} r_{2_1}\right)\left(\frac{\partial^2}{\partial t^2} r_{2_2}\right) - \left(\frac{\partial^2}{\partial t^2} r_{2_1}\right)\left(\frac{\partial}{\partial t} r_{2_2}\right)}{\left(\left(\frac{\partial}{\partial t} r_{2_1}\right)^2 + \left(\frac{\partial}{\partial t} r_{2_2}\right)^2\right)^{\left(\frac{3}{2}\right)}} \tag{4}$$

The common arc lengths $L_1 = L_2$ of the centrodes in the moment $t = \tau$ are

$$L_1 := \int_0^{\tau} \sqrt{R_1^{\,2} + \left(\frac{\partial}{\partial t} R_1\right)^2}\, dt \qquad L_2 := \int_0^{\tau} \sqrt{\left(\frac{\partial}{\partial t} r_{2_1}\right)^2 + \left(\frac{\partial}{\partial t} r_{2_1}\right)^2}\, dt \tag{5}$$

The sum of the phase of motion of the centrodes is shown in Fig. 4.

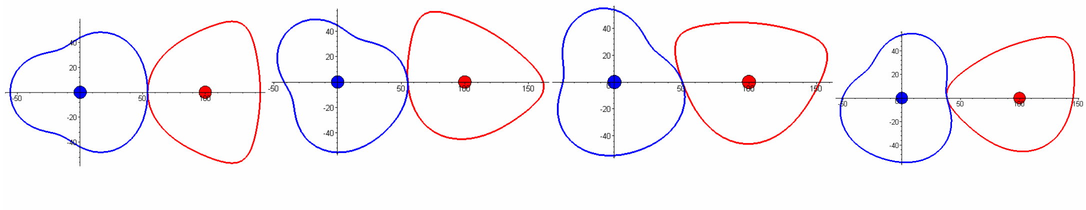


Figure 4

The angle of tangent lines for the pitch curves relative to the horizontal in original positions of the centrodes is calculated by

$$\mu_i := \arctan\left(\frac{\frac{\partial}{\partial t} r_{i_2}}{\frac{\partial}{\partial t} r_{i_1}}\right) \qquad i = 1, 2 \tag{6}$$

3. The basic racks of the gears

Let the common number of the teeth in the gears be $z_1 = z_2 = z$. The total length is $L_1 = L_2$ of the pitch curves by formulae (5) in the case of $\tau = 2\pi$. The module of involute gears calculated by

$$m = \frac{L_i}{z.\pi}\ , \ i = 1,2 \tag{7}$$

The system of equations of the standard basic rack tooth profile with module m = 2 is

$$S(u) := \begin{cases} \dfrac{u}{\tan(\alpha)} & u < h_1 \tan(\alpha) \\ h_1 & u < \pi - h_1 \tan(\alpha) \\ \dfrac{\pi - u}{\tan(\alpha)} & u < \pi + h_2 \tan(\alpha) \\ -h_2 & u < 2\,\pi - h_2 \tan(\alpha) \\ \dfrac{u - 2\,\pi}{\tan(\alpha)} & \mathit{otherwise} \end{cases} \tag{8}$$

where the pressure angle is α, the factor of addendum is h_1, the factor of dedendum is h_2, and the root fillet radius is 0. While every period of (8) can be written in the general form S(u) = k.u – c, all parts according to 2π periodical function system (8) can be explanted with the Fourier coefficients in the general form

$$A_j := \frac{\int_0^{2\pi} (k\,u - c)\sin(j\,u)\,du}{\pi}, \qquad B_j := \frac{\int_0^{2\pi} (k\,u - c)\cos(j\,u)\,du}{\pi} \tag{9}$$

The expanded and simplified integrals (9) can be evaluated in closed forms by

$$\begin{aligned} A_j &:= \frac{(c\,j - 2\,k\,j\,\pi)\cos(2\,\pi\,j)}{j^2\,\pi} + \frac{-c\,j + k\sin(2\,\pi\,j)}{j^2\,\pi} \\ B_j &:= \frac{(-c\,j + 2\,k\,j\,\pi)\sin(2\,\pi\,j)}{j^2\,\pi} + \frac{-k + k\cos(2\,\pi\,j)}{j^2\,\pi} \end{aligned} \tag{10}$$

The Fourier series of the standard rack profile in order M_0 by module m = 2 is

$$Z := \frac{B_0}{2} + \left(\sum_{i=1}^{Mo} \left(A_{2\,i-1}\sin((2\,i-1)\,u) + B_{2\,i}\cos(2\,i\,u) \right) \right) \tag{11}$$

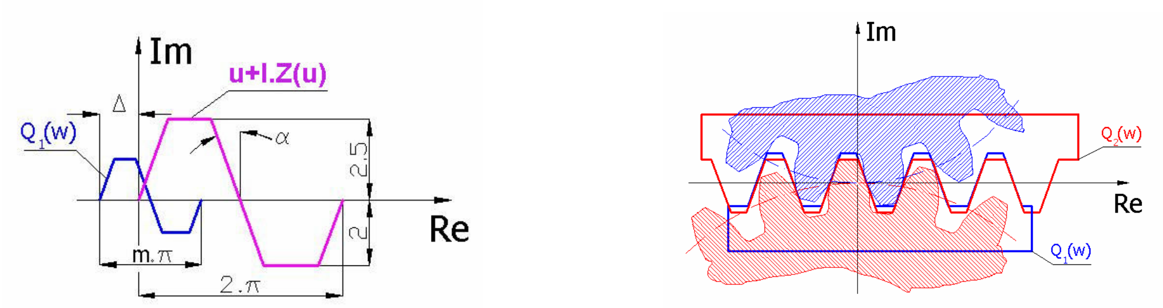
If the actual module is by (7) the concrete Fourier series by substituting $u = \frac{2.w}{m}$ is

$$H(w) = \frac{2}{m}.Z(u = \frac{2.w}{m}) \tag{12}$$

where w is the formal parameter of the rack. The complex formulae's of the racks for gears with pitch curves (2) are

$$Q_1(w) = w + \Delta + I.H(w) \quad , \quad Q_2(w) = \overline{Q_1(w)} - \frac{m.\pi}{2} \tag{13}$$

$I = \sqrt{-1}$ and the sign of over case is the complex conjugate. The representation of racks by m = 2 and by concrete value (7) of module m are shown in Fig. 5 a/ and b/.



a/ b/

Figure 5

4. The profiles of teeths

If the rotation angle of the driving gear (1) is τ, the angle of rotation of the driven gear (2) is $\Phi(\tau)$, the co-ordinate systems Σ_1 and Σ_2 are rigidly connected to the axes 1 and 2. The racks 1 and 2 by Eq. (13) are pure rolling on the centrodes No.1 and 2. by Eq. (3). The complex co-ordinates of rack points by parameter w is written by

$$T_1(w, \tau) := (Q_1(w) - L_1(\tau))\,\mathbf{e}^{(I\mu_1(\tau))} + R_1(\tau)\,(\cos(\tau) + I\sin(\tau))$$

$$T_2(w, \tau) := (Q_2(w) - L_2(\tau))\,\mathbf{e}^{(I\mu_2(\tau))} + R_2(\tau)\,(\cos(\Phi(\tau)) + I\sin(\Phi(\tau))) \tag{14}$$

The centrode No.1 have convex and concave parts too. The τ'_i parameters of inflections points are calculated from $k_1(\tau'_i) = 0$ by the numerical solution of Eq. (4) The generating rack (13) is separated into the parts Q'_1, Q'_2, … by the restrictions

$$\begin{gathered} Q'_1 = Q_1(0 \le w \le \tau'_1),\ \ Q'_2(\tau'_1 \le w \le \tau'_2), \\ \dots, \\ Q'_j(\tau'_{j-1} \le w \le \tau'_j),\ \dots,\ Q'_m(\tau'_{m-1} \le w \le 2\pi)) \end{gathered} \tag{15}$$

The discrete rack parts positions for cases $\tau = 0, \delta, 2\delta, \dots, n\delta, \dots, 2\pi$ are shown in Fig. 6.
If the centrode is convex on the actual part on the pitch curve the effective rack cut out the allowances from the spaces of teeth (from the outer side of the pitch curve). In case of a concave part the centrode build up the rack of the body of the teeth (from inner side of the pitch curve).
The application into a CAD system can be used with the suitable set commands “SUBSTRACT” and “UNION” for the plates which represented the partial rack elements by (15).

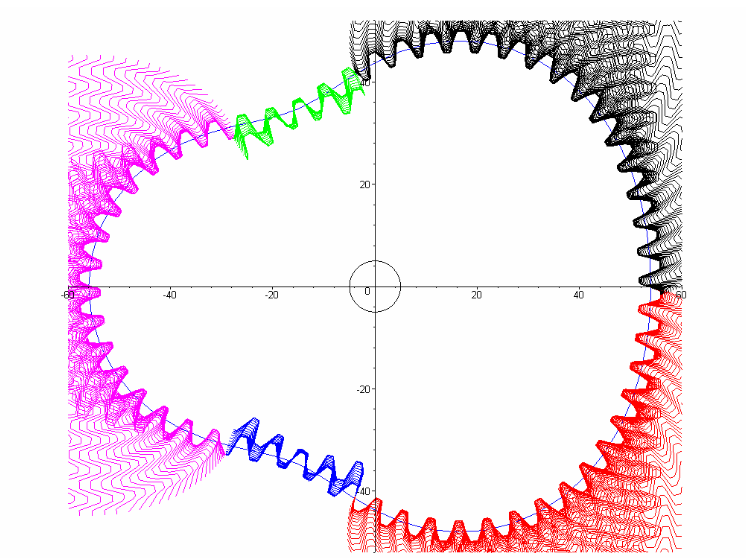
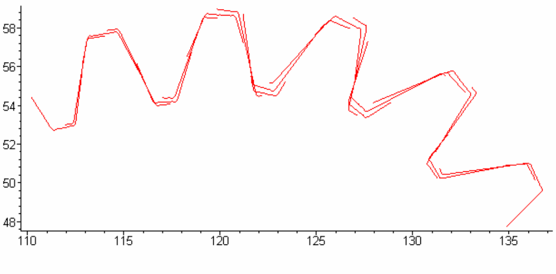


Figure 6

Additional restriction of generating rack parts can be made by the following:
Let the length of rolled pitch curve $L_i(w)$. The actual rack part is $Q_i(w)$ according (13), where

$$L_i - \frac{m.\pi}{4} \le w < L_i + \frac{m.\pi}{4} \qquad (16)$$

The CAD application of formulae (15) and (16) by (14) is shown in Fig. 7 and 8.



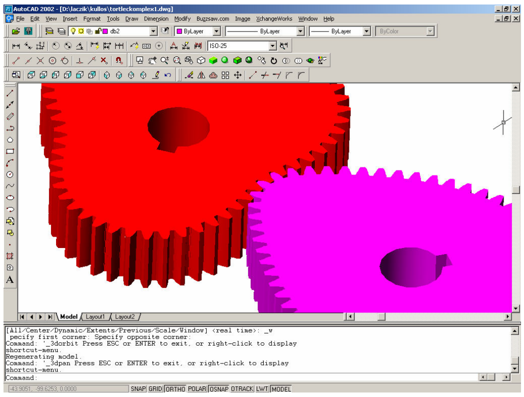


Figure 7 Figure 8

The analytic derivation of the correct involute and the undercut part of the NCGs teeth is given in Reference [14]. The method is theoretically clear but they application of it in general case is rather cumbrous.

5. Summary

The study makes a review of principles design and manufacturing of non-circular gears. The contribution delineates the basics of analytic formulae of pitch curves for a given transfer function. The article presents the method for modeling of the basic rack by complex Fourier series and techniques of CAD modeling of the gears.

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