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John D. Reiff, M.Sc.[1]

A Procedure for Calculation of Torque Specifications for Bolted Joints with Prevailing Torque

ABSTRACT: This paper presents procedures developed for the calculation of the coefficient of friction of bolt/nut assemblies and for the calculation of torque specifications, which include the case where the fasteners have prevailing torque

KEYWORDS: fastener torque, prevailing torque, torque calculation, head finish, thread finish

Nomenclature

The following is a list of the nomenclature used. Most terms are consistent with the nomenclature used in the VDI 2230 procedure.

A_0	Smallest cross-section area of bolt
A_S	Effective tensile stress cross-section of the bolt thread per ISO 898-1
D_0	Outside diameter of bolt at the smallest cross-section, A_0 (smaller of D_S or D_T)
D_2	Pitch diameter of bolt thread
D_3	Minor diameter of bolt thread
D_{km}	Effective diameter for friction at the contact of the head of the driven fastener
D_S	Diameter at stress cross-section A_S
D_T	Shank diameter of bolt neck
D_W	Outside diameter of the contact area under the head of the driven fastener
F_M	Assembly preload, bolt tensile load at tightening
$F_{M,\nu}$	Assembly preload, bolt tensile load at which the equivalent stress is $\nu R_{p,0.2}$
$F_{M,MIN}$	Minimum assembly preload expected from tightening to the specified torque
$F_{M,MAX}$	Maximum assembly preload expected from tightening to the specified torque
M_A	Assembly input torque
$M_{A,MIN}$	Maximum assembly input torque
$M_{A,MAX}$	Minimum assembly input torque
$M_{A,PRE}$	Assembly prevailing torque
M_G	Assembly thread torque, moment in the bolt neck
P	Pitch of the bolt thread
$R_{p,0.2}$	0.2 % proof stress of bolt material per ISO 898-1
d_i	Inside diameter of hollow bolt
d_h	Inside diameter of the contact area under the head of the driven fastener
β_{th}	Half flank angle of the bolt thread ($\pi/6$ for ISO thread)
μ_G	Coefficient of friction between bolt and nut thread
$\mu_{G,MIN}$	Minimum coefficient of friction between bolt and nut thread

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[1] General Motors Corporation, 30001 Van Dyke Ave., Warren, MI.

$\mu_{G,MAX}$	Maximum coefficient of friction between bolt and nut thread
μ_K	Coefficient of friction at the contact of the driven fastener head
$\mu_{K,MIN}$	Minimum coefficient of friction at the contact of the driven fastener head
$\mu_{K,MAX}$	Maximum coefficient of friction at the contact of the driven fastener head
μ_{ges}	Coefficient of friction; both head and thread friction are equal
σ_e	Equivalent stress
σ_m	Tensile stress
τ	Torsional stress
ν	Degree of exploitation of bolt yield stress desired at maximum assembly condition

Introduction

Tightening tests and methods for calculation of the coefficient of friction at the driven fastener's bearing surface and at the thread contact area are specified in the ISO standard for Fasteners – Torque/Clamp Force Testing (16047) and in the German national standard, Determination of Coefficient of Friction of Bolt/Nut Assemblies Under Specified Conditions (DIN 946). A method for the calculation of torque specifications which requires these friction coefficients is described in the well-known procedure, Systematic Calculation of High Duty Bolted Joints (VDI 2230). These procedures and calculation methods are developed and applicable only for the case where the fasteners have no significant prevailing torque. Prevailing torque is the torque required to turn the driven fastener before any clamping force (or bolt tension) is generated.

In the automobile industry many critical attachments are designed with fasteners that include prevailing torque features, such as all-metal prevailing torque nuts. The error resulting from the application of the standards (referred to above) for calculation of fastener friction and torque specifications has been determined to be of significant magnitude. Therefore, new calculation procedures have been developed and are described in this paper. For the case with no prevailing torque, these same equations can be used but with the value of prevailing torque set to zero.

Theory

Equations for Calculation of Friction

The following equations are developed for calculating the coefficient of head friction (μ_K) and the coefficient of thread friction (μ_G) from data measured during a nut/bolt tightening process. During the tightening process, measured values include input torque (M_A), thread torque (M_G), and bolt tension (F_M).

A mathematical model of the tightening process has been developed by Motosh [1] and is modified here to include the prevailing torque term ($M_{A,PRE}$):

$$F_M = \frac{\left(M_A - M_{A,PRE}\right)}{\left(\frac{P}{2\pi} + \frac{\mu_G D_2}{2\cos\beta_{th}} + \frac{\mu_K D_{km}}{2}\right)} \quad (1)$$

Where:

$$D_{km} = \frac{D_w + d_h}{2} \tag{2}$$

This equation is rearranged to solve for input torque, which is reacted by the torque under the head of the driven fastener (head torque) and the torque at the contact of the nut and bolt threads (thread torque):

$$\underbrace{M_A}_{\text{input torque}} = \underbrace{M_{A,PRE} + F_M\left(\frac{P}{2\pi} + \frac{\mu_G D_2}{2\cos\beta_{th}}\right)}_{\text{thread torque}} + \underbrace{F_M\left(\frac{\mu_K D_{km}}{2}\right)}_{\text{head torque}} \tag{3}$$

Thread torque includes:

$$M_G = M_{A,PRE} + F_M\left(\frac{P}{2\pi} + \frac{\mu_G D_2}{2\cos\beta_{th}}\right) \tag{4}$$

And the head torque is the difference of input torque minus thread torque:

$$M_A - M_G = F_M\left(\frac{\mu_K D_{km}}{2}\right) \tag{5}$$

Equation 4 is rearranged to solve for thread friction:

$$\mu_G = \frac{2\cos\beta_{th}}{D_2}\left(\frac{M_G - M_{A,PRE}}{F_M} - \frac{P}{2\pi}\right) \tag{6}$$

Equation 5 is rearranged to solve for head friction:

$$\mu_K = \frac{2(M_A - M_G)}{D_{km}F_M} \tag{7}$$

And finally, if head friction and thread friction are assumed to be equal, then μ_{ges} is substituted for μ_G and μ_K in Eq 3, and the equation is rearranged to solve for friction:

$$\mu_{ges} = \frac{\left(\frac{M_A - M_{A,PRE}}{F_M} - \frac{P}{2\pi}\right)}{\left(\frac{D_2}{2\cos\beta_{th}} + \frac{D_{km}}{2}\right)} \tag{8}$$

Equations 6 and 7 are used to calculate the coefficients of head and thread friction from the measurement of prevailing torque, input torque, and thread torque, at a selected value of bolt tension during assembly of fasteners in a laboratory test.

Equations for Calculation of Torque Specification

The following equations are developed for calculating a torque specification, utilizing the coefficients of head and thread friction calculated using equations above. During the assembly of the fasteners, the bolt shank and threaded section are stressed in tension and additionally in shear due to the applied torque. The equations are developed to allow for the calculation of an upper torque specification limit that will result in a desired maximum equivalent stress in the bolt shank due to the tension and shear combined stresses. The minimum torque specification is calculated to result in a specified tolerance so that the tightening process will be statistically capable for a selected tightening tool. For example a torque specification tolerance of ±15 % might be required to have a capable process with a selected mechanical clutch shutoff tool.

The equivalent stress in the bolt due to the tensile stress and the torsional stress from the maximum distortion energy theory of failure) is:

$$\sigma_e = \sqrt{\sigma_M^2 + 3\tau^2} \tag{9}$$

The desired magnitude for the maximum equivalent stress resulting from tightening is:

$$\sigma_e = \upsilon R_{p,0.2} \tag{10}$$

The tensile stress is:

$$\sigma_M = \frac{F_{M,\nu}}{A_0} \tag{11}$$

And the torsional stress is:

$$\tau = \frac{16 M_G D_0}{\pi\left(D_O^4 - d_i^4\right)} \tag{12}$$

Substituting Eqs 10, 11, 12 and 4 into Eq 9 and solving for $F_{M,\nu}$ yields the following equation. Here, the value $F_{M,\nu}$ is the allowable magnitude for the bolt preload such that the equivalent stress is $\nu R_{p,0.2}$ for any value of thread friction, μ_G.

$$F_{M,\nu} = -6M_{A,PRE}K_1K_2 + \frac{\sqrt{\left(6M_{A,PRE}K_1K_2\right)^2 - 4\left(1+3K_1K_2{}^2\right)\left(3M_{A,PRE}{}^2K_1 - K_3\right)}}{2\left(1+3K_1K_2{}^2\right)} \tag{13}$$

Where:

$$K_1 = \left[\frac{4D_0}{\left(D_0{}^2 + d_i{}^2\right)}\right]^2, \quad K_2 = \left[\frac{P}{2\pi} + \frac{\mu_G D_2}{2\cos\beta_{th}}\right], \text{ and } K_3 = \left[\frac{\nu R_{P,0.2}\pi\left(D_0{}^2 - d_i{}^2\right)}{4}\right]^2$$

For the case where the minimum bolt cross-section is the threaded section:

$$D_0 = \frac{D_2 + D_3}{2} \tag{14}$$

And for the case where the minimum bolt cross section is the shank:

$$D_0 = D_T \tag{15}$$

If the upper and lower limits of the torque specification are $M_{A,MAX}$ and $M_{A,MIN}$, respectively, then the torque specification tolerance is:

$$M_{A,TOL} = \frac{\left(M_{A,MAX} - M_{A,MIN}\right)}{2} \tag{16}$$

The nominal of the torque specification is:

$$M_{A,NOM} = \frac{\left(M_{A,MAX} + M_{A,MIN}\right)}{2} \tag{17}$$

The tolerance in terms of a percentage of nominal is:

$$M_{A,TOL\%} = \frac{M_{A,TOL}}{M_{A,NOM}} \tag{18}$$

In order to calculate the maximum bolt preload ($F_{M,MAX}$) so that the equivalent stress does not exceed the value $\nu R_{p,0.2}$, the minimum value of thread friction is substituted into Eq 13:

$$F_{M,MAX} = -6M_{A,PRE}K_1K_2 + \frac{\sqrt{\left(6M_{A,PRE}K_1K_2\right)^2 - 4\left(1+3K_1K_2^{\,2}\right)\left(3M_{A,PRE}^{\,2}K_1 - K_3\right)}}{2\left(1+3K_1K_2^{\,2}\right)} \tag{19}$$

Where:

$$K_1 = \left[\frac{4D_0}{\left(D_0^{\,2} + d_i^{\,2}\right)}\right]^2, \quad K_2 = \left[\frac{P}{2\pi} + \frac{\mu_{G,MIN}D_2}{2\cos\beta_{th}}\right], \text{ and } K_3 = \left[\frac{\nu R_{P,0.2}\pi\left(D_0^{\,2} - d_i^{\,2}\right)}{4}\right]^2$$

The maximum torque is calculated by substituting Eq 16 into Eq 3 with minimum values of head and thread friction:

$$M_{A,MAX} = M_{A,PRE} + F_{M,MAX}\left[\frac{P}{2\pi} + \frac{\mu_{G,MIN}D_2}{2\cos\beta_{th}} + \frac{\mu_{K,MIN}D_{km}}{2}\right] \tag{20}$$

The minimum torque is calculated from the maximum based on the desired torque tolerance:

$$M_{A,MIN} = M_{A,MAX}\frac{\left(1 - M_{A,TOL\%}\right)}{\left(1 + M_{A,TOL\%}\right)} \tag{21}$$

And finally, the bolt minimum preload when assembled to the minimum torque is calculated by substituting $M_{A,MIN}$ into Eq 1 with maximum values of head and thread friction:

$$F_{M,MIN} = \frac{\left(M_{A,MIN} - M_{A,PRE}\right)}{\frac{P}{2\pi} + \frac{\mu_{G,MAX} D_2}{2\cos\beta_{th}} + \frac{\mu_{K,MAX} D_{km}}{2}} \tag{22}$$

Discussion

Error Due to Misapplication of Standard Equations

The equations used for the calculation of friction in ISO 16047 [1] and in the German national standard DIN 946 [2] are:

$$\mu_G = \frac{2\cos\beta_{th}}{D_2}\left(\frac{M_G}{F_M} - \frac{P}{2\pi}\right) \tag{23}$$

and:

$$\mu_K = \frac{2\left(M_A - M_G\right)}{D_{km} F_M} \tag{24}$$

The root of these equations is the long form equation that describes the relationship between torque and tension during the assembly of a fastener:

$$F_M = \frac{M_A}{\left(\frac{P}{2\pi} + \frac{\mu_G D_2}{2\cos\beta_{th}} + \frac{\mu_K D_{km}}{2}\right)} \tag{25}$$

For a given bolt/nut assembly:

$$\frac{1}{\left(\frac{P}{2\pi} + \frac{\mu_G D_2}{2\cos\beta_{th}} + \frac{\mu_K D_{km}}{2}\right)} = K \ \text{(Constant)}$$

and Eq 25 reduces to:

$$F_M = KM_A \tag{26}$$

This is an equation for a line that passes though the origin of the input torque versus bolt tension graph. The point here is explained in Fig. 1. The line labeled "actual" represents the torque/tension relationship for a bolt/nut assembly with prevailing torque, $M_{A,PRE}$. If the friction is calculated based on values of M_A and M_G at the bolt tension $F_{M,1}$ using Eqs 23 and 24, then the analytical description of the torque/tension relationship, based on Eq 25, is shown by the line labeled "analytical." As shown in the figure, the calculated value of bolt tension at M_A is $F_{M,3}$, and the actual value of bolt tension is $F_{M,2}$, resulting in the error shown. Also, note that the error is zero exactly at the value of tension at which the head and thread friction were calculated.

For some top-lock all-metal prevailing torque nuts, the error between the actual and calculated values of $F_{M,,MIN}$ is approximately 15 %.

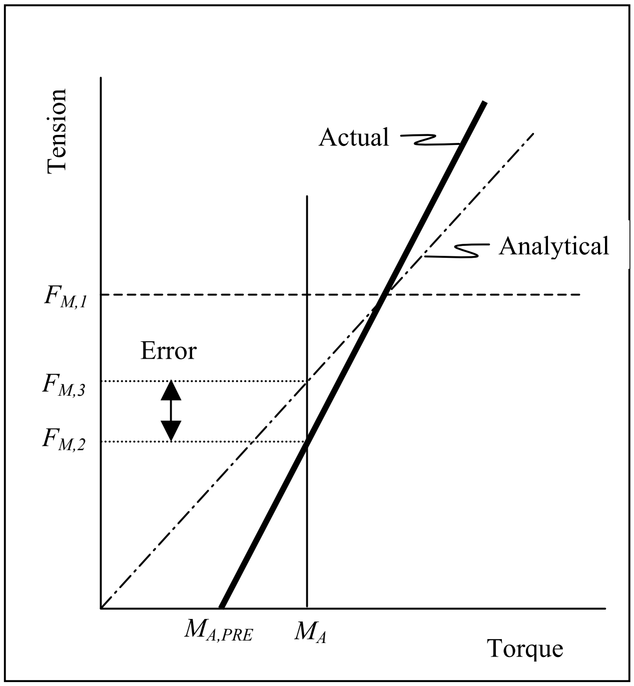

FIG. 1—*Diagram of error using standard method.*

Discussion of Method for Torque Specification Calculation

Figure 2 shows the process described above for the calculation of torque specifications. The curved line labeled $F_{M,MAX}$ shows the values of bolt preload that result in the desired equivalent stress $\nu R_{P0.2}$ for various values of μ_G. The two bold lines that intersect the torque-axis at $M_{A,PRE}$ describe the limits of the bolt/nut tightening process, where the under head friction varies from $\mu_{K,MIN}$ to $\mu_{K,MAX}$, and the thread friction varies from $\mu_{G,MIN}$ to $\mu_{G,MAX}$. The maximum of the torque specification $M_{A,MAX}$ is at the intersection of the F_M line, and the line that describes the bolt/nut tightening process is at the minimum values of friction. So, at the maximum of the torque specification, and with fasteners that have minimum friction at both the head and thread, the bolt equivalent stress is at the desired maximum value. $M_{A,MIN}$ is calculated from $M_{A,MAX}$ to achieve a desired torque tolerance for process capability. And finally, $F_{M,MIN}$ is at the intersection of $M_{A,MIN}$, and the line that describes the bolt/nut tightening process is at the maximum values of friction.

Conclusion

Equations 20 and 21 can be used for the calculation of torque specifications for the case of fasteners with prevailing torque. The resulting maximum and minimum bolt preload is calculated with Eqs 19 and 22. When these equations are used, the fastener head and thread friction must be calculated per Eqs 6 and 7. The use of these equations can result in a 15 % improvement in the accuracy of torque specification calculation, when compared to the results of calculations that do not consider the prevailing torque.

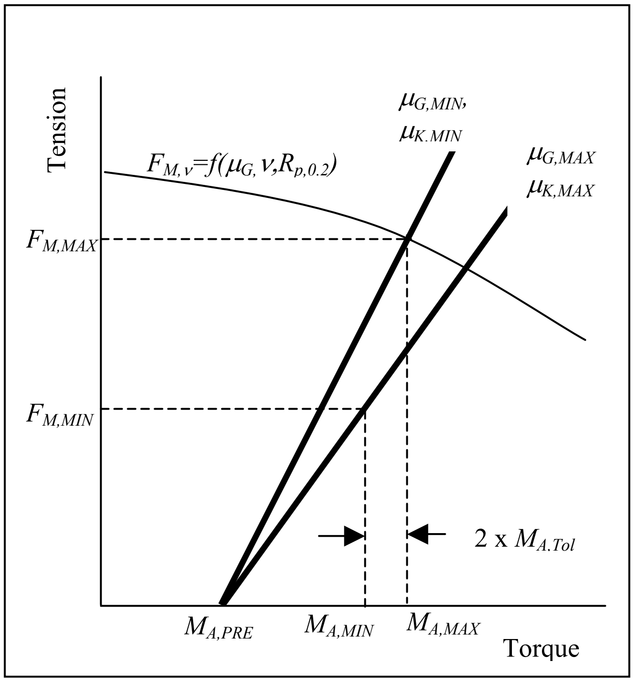

FIG. 2—*Diagram of procedure for calculation of torque specification.*

References

[1] Motosh, N., "Development of Design Charts for Bolts Preloaded up to the Plastic Range," *J. Eng. Ind.*, August 1976.