Formulas for the
Calculation of the Engagement Length
of Screws

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1 Introduction
The calculation of the screw penetration in a tapped blind hole thread must assure that in the case of overload it is sufficient to result in the break of the screw rather than the stripping off of the thread. Furthermore, the safety of the stress cross section \( A_s \) must be equal to the safety of the load-bearing threads. One must consider the tensile strength of the paired materials. Thus it is necessary that the tensile strength of the screw bolt is at the maximum, whereas the nut material has the minimum tensile strength. To calculate the screw break load one multiplies the stress cross section \( A_s \) with the tensile strength of the screw.

For over 115 experiments bolts with a tolerance of 6g and nuts with a tolerance of 6H were manufactured. The tensions and the calculated shearing tension factor \( \beta_{B,M} \) shown in the experiments included non-quantifiable influences like:
- Shear off stress of the engaged threads;
- Bending stress of the engaged threads;
- Influence of the pairing 6H/6g for small threads;
- Effect of notches (Stress concentration factor \( \alpha_k \) and Support Factor \( \nu \)) in the thread part.

To differentiate real shear off stress from the ersatz tension calculated above we call this Shearing Tension. It is a fatal error to assume that for screw bolts one can use in calculations the shear off stress obtained transverse to the direction of force (minor diameter). Measurements at the University of Siegen demonstrated that Shearing Tensions of threads in the direction of force are of a different magnitude than shear off stress in the minor diameter.

With larger threads and tolerances one finds larger flank clearance of the non-bearing flanks; they reduce the shearing area of the stripping off cylinder and result in a calculated screw penetration that is too small. One has to counter-balance the tolerances, i.e., increase the screw penetration.

2 Symbols and Notations
The following general symbols are used

- \( F = R_m \cdot A_s \) \([\text{N}]\) Screw break load
- \( F_{\text{max}} \) \([\text{N}]\) Max. General Forces and Preload
- \( \alpha_{B,M} \) \([-\text{]}\) Material factors of screw or nut
- \( \beta_{B,M} \) \([-\text{]}\) Shearing tension factors
- \( \alpha \) \([\text{°}]\) Flank angle
- \( R_{m,B,M} \) \([\text{N/mm}^2]\) Tensile strength of the screw or nut material
- \( R_{p,B,M} \) \([\text{N/mm}^2]\) Yielding point of the screw or nut material
- \( \tau_{m,B,M} = \beta_{B,M} \cdot R_{m,B,M} \) \([\text{N/mm}^2]\) Shearing tension limit of the screw or nut
- \( \tau_{p,B,M} = \beta_{B,M} \cdot R_{p,B,M} \) \([\text{N/mm}^2]\) Screw or nut shear tension limit at the yielding point
- \( \tau_{B,M,\text{vor}} \) \([\text{N/mm}^2]\) Existing screw or nut shearing tension
- \( D_1 \) \([\text{mm}]\) Minor diameter of nut
- \( D_2 \) \([\text{mm}]\) Pitch diameter of nut
- \( d \) \([\text{mm}]\) Major diameter
- \( d_2 \) \([\text{mm}]\) Pitch diameter of screw
- \( d_3 \) \([\text{mm}]\) Minor diameter of screw
- \( d_t \) \([\text{mm}]\) Diameter of strip off cylinder
- \( m \) \([\text{mm}]\) Length of thread engagement (LE)
- \( m_{\text{tol}} \) \([\text{mm}]\) Additional length of the toleration
- \( m_{\text{rfl}} \) \([\text{mm}]\) Min. length of thread engagement
- \( A_s \) \([\text{mm}^2]\) Stress cross section of the screw
- \( A_t \) \([\text{mm}^2]\) Total surface of strip off cylinder
- \( A_{B,M} \) \([\text{mm}^2]\) Surface of strip off cylinder of screw or nut thread
- \( P \) \([\text{mm}]\) Pitch
- \( D_2_{\text{max}} \) \([\text{mm}]\) Pitch diameter of the nut with toleration
- \( d_5_{\text{min}} \) \([\text{mm}]\) Pitch diameter of the screw with toleration
- \( s \) \([\text{mm}]\) Pass of toleration at the pitch diameter
- \( w \) \([-\text{]}\) Number of winding
3 Thread Engagement
To calculate the tensile strength of the paired material one must consider.

3.1 Up to Tolerance of 6H/6g and M30
The shearing tension limit of the material:

Bolt, screw
\[ \tau_{mb} = \beta_B \cdot R_{mb} \]  \hspace{1cm} (1a)

Nut
\[ \tau_{mm} = \beta_M \cdot R_{mm} \]  \hspace{1cm} (1b)

Screw break load
\[ F = R_{mb} \cdot A_s \]  \hspace{1cm} (2)

Material- factor for bolt
\[ \alpha_B = \frac{\tau_{mm}}{\tau_{mb}} \]  \hspace{1cm} (3a)

Material factor for nut
\[ \alpha_M = 1 - \alpha_B = \frac{\tau_{mm}}{\tau_{mb} + \tau_{mm}} \]  \hspace{1cm} (3b)

Strip off diameter
\[ d_t = d_2 + (0.5 - \alpha_B) \cdot P / \tan 30^\circ \]  \hspace{1cm} (4)

In case if \( d_t < D_1 \)
\[ d_t = D_1 \]

In case if \( d_t > d \)
\[ d_t = d \]

Total surface of strip off cylinder
\[ A_t = F \cdot \left(\frac{1}{\tau_{mb}} + \frac{1}{\tau_{mm}}\right) \]  \hspace{1cm} (5)

Length of thread engagement (LE)
\[ m = A_t / (d_t \cdot \pi) \]  \hspace{1cm} (6)

Attention:
The tip of the screw thread is not included in the calculation of the engagement length.

3.2 At higher thread tolerance and more than M30
To the calculated length of engagement \( m \) must be added the tolerance. For following calculation steps are necessary

Number of windings:
\[ w = m / P \]  \hspace{1cm} (7)

Pass of toleration at the pitch diameter:
\[ s = (D_2 \text{ max} - d_2 \text{ min}) \cdot \tan 30^\circ \]  \hspace{1cm} (8)

Additional length of toleration:
\[ m_{hol} = w \cdot s \]  \hspace{1cm} (9)

Min. length of thread engagement
\[ m_{erf} = m + m_{hol} \]  \hspace{1cm} (10)

4 Calculation of shearing tension
4.1 Up to Tolerance of 6H/6g
With formula (1a) and (1b) calculate formula (3a), than formula (4)

Total surface of strip off cylinder
\[ A_t = d_t \cdot \pi \cdot m \]  \hspace{1cm} (11)

Part of strip off cylinder of bolt:
\[ A_{tB} = \alpha_B \cdot A_t = F / \tau_{tB} \]  \hspace{1cm} (12a)

Part of strip off cylinder of nut:
\[ A_{tM} = \alpha_M \cdot A_t = F / \tau_{tM} \]  \hspace{1cm} (12b)

With the max. general force \( F_{\text{max}} \) one will find
Existing screw shearing tension.
\[ \tau_{B \text{ vor}} = F_{\text{max}} / A_{tB} \]  \hspace{1cm} (13a)

Existing nut shearing tension.
\[ \tau_{M \text{ vor}} = F_{\text{max}} / A_{tM} \]  \hspace{1cm} (13b)

To calculate the safety rate within the threads to the yield point one uses formula (1a) and (1b) with the shear tension limit at the yield point \( \tau_{p,B,M} = \beta_{B,M} \cdot R_{pB,M} \)

Safety rate for the threads of the bolt
\[ \nu_B = \tau_{pB} / \tau_{B \text{ vor}} \]  \hspace{1cm} (14a)

Safety rate for the threads of the nut
\[ \nu_M = \tau_{pM} / \tau_{M \text{ vor}} \]  \hspace{1cm} (14b)

4.2 At higher Tolerance and more than M30
From the real engagement length \( m_{\text{vor}} \) the tolerance must be subtracted. The calculation is as follows

Number of windings
\[ w = m_{\text{vor}} / P \]  \hspace{1cm} (15)

With the tolerance per winding with formula (8)

To subtract engagement length
\[ m_{\text{hol}} = w \cdot s \]  \hspace{1cm} (9)

Calculated engagement length
\[ m_{\text{ech}} = m_{\text{vor}} - m_{\text{hol}} \]  \hspace{1cm} (16)

With formula (4) the Calculated surface of strip off cylinder is
\[ A_t = d_t \cdot \pi \cdot m_{\text{ech}} \]  \hspace{1cm} (17)

Continue with formula(12) to (14)
5 Shearing tension factors

Material for Screws:
Bolts of the strength grades according to DIN EN ISO 898-1 $\beta_B = \frac{1}{\sqrt{3}}$
Bolts of the strength grades according to DIN EN ISO 3596-1 $\beta_B = 0.7$
Bolts of steel in Ferritic $\beta_B = \frac{1}{\sqrt{3}}$
Bolts of stainless steel in Martensitic $\beta_B = \frac{1}{\sqrt{3}}$
Bolts of stainless steel in Austenitic $\beta_B = 0.7$

Material for Nuts:
Steel in Ferritic $\beta_M = \frac{1}{\sqrt{3}}$
Stainless steel in Martensitic $\beta_M = \frac{1}{\sqrt{3}}$
Stainless steel in Austenitic $\beta_B = 0.7$
Cast steel (GS) $\beta_M = \frac{1}{\sqrt{3}}$
Cast iron (GJL) $\beta_M = 0.9$
Cast Iron (GJS) $\beta_M = 0.7$
Aluminium $\beta_M = 0.44$